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Determination of linear characteristics of rotor mounting groups under load

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Introduction. The paper considers analytical studies on the "rotor – gapped-type support" dynamic system under process loading. The research objective is to obtain expressions for determining the equivalent stiffness characteristics of the system.

Materials and Methods. A rotor rotating in the elastic gapped-type supports is considered. A dynamic model that enables to consider the problem of determining the linear equivalent stiffness characteristics of mounting groups is proposed. To solve the problem, a system of differential equations is compiled, and a detailed analysis is performed.

Results. From the obtained dynamic equations of the system in question, we can calculate the static angular deviation of the rotor pins due to the action of the load. The proposed expressions for determining equivalent stiffness characteristics testify that it is possible to study the rotor dynamics as on the linear elastic supports with the above parameters. The obtained system of equations is analyzed, and all special cases of applying the first approximation formulas for equivalent stiffness of the rotor mounting groups are listed.

Discussion and Conclusions. The results obtained make it possible to study many dynamic processes on the basis of linear differential equations considering the nonlinear properties of the system. For shavers used in the production of leather materials, the determination of rotor vibrations in the horizontal plane provides the quality and accuracy of operations.

Keywords*:* mechanical engineering, rotor, dynamic rotor equipment, rotor balance, rotor mounting groups, linear equivalent stiffness parameters of rotor units, rotor operating modes.

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Introduction. Technological progress imposes new, higher requirements for the quality of engineering products. Mainly, it is about high reliability, durability of machines, their productivity and safety. All these parameters should be taken into account and calculated at the design stage.

From the point of view of production processes, the dynamic rotor equipment that provides the continuity of the process is of particular importance [1-3]. In [4], structural designs and the principle of operation of various rotary machines are presented; patent searches for each type are performed. In [5], the nature of the dependence of equivalent stiffness on the frequency of oscillations for a specific model is considered; the dependence of the critical frequencies on the equivalent stiffness of the supports is given. In [6], the authors consider in detail the issues of vibration control, vibro-adjustment work and prevention of increased vibration of rotor machines, and equilibrium of rotors. In addition, vibration sources are listed here and basic information from oscillation theory is provided. The problems of the dynamics of a rigid unbalanced rotor with four degrees of freedom are discussed in [7–9]. The study [10] is based on the assumption that bearing reactions are quasilinear forces with cubic nonlinearity. With this in mind, the effect of the radial clearance on the movement in space of a dynamically unbalanced rotor under the impact of internal friction forces is considered. In [11], the interrelation of transverse and torsional vibrations arising under the rotation of the centrifuge

rotor is shown. A linearized mathematical model of the rotor in elastic supports is developed due to the impact of transverse and torsional vibrations.

The literature review suggests that the study on the oscillatory process and the corresponding parameters of linear mechanical systems is of interest. In this paper, the rotor, in particular, its gapped-type supports, is considered. The study objective is to determine the values under load of linear equivalent stiffness parameters of the named mounting groups.

Materials and Methods. A dynamic model that provides for rotation in elastic gapped-type supports was used as a rotor (Fig. 1).

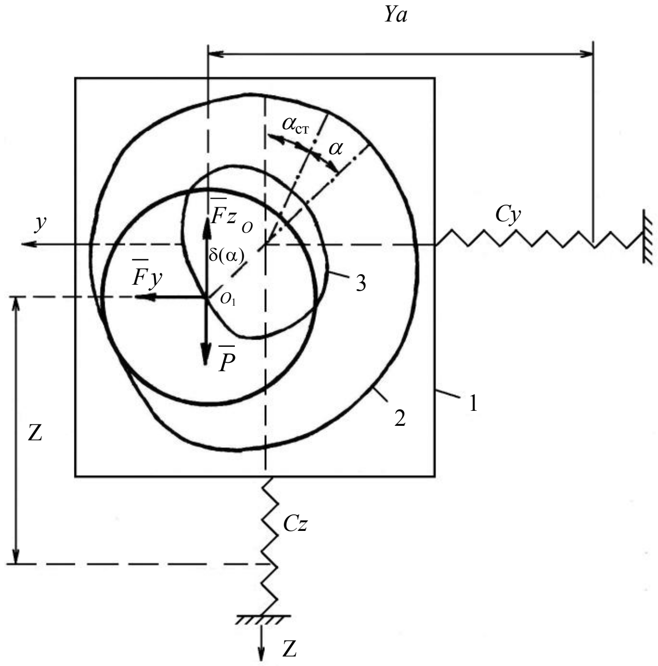

Fig. 1. Dynamic model of a rotor that rotates in elastic gapped-type supports

In Fig. 1, the following designations are adopted: 1 is rotor support; 2 is the contact boundary of the support with the rotor axle; 3 is the curve of relative motion of the rotor axle center; m is the rotor mass; β (α) is the radial clearance in the rotor bearings (represented by the angular displacement function of its journals from the vertical guides); C_y, C_z are the general stiffness of the casings of the mounting groups in the horizontal and vertical guides, respectively; $y_{\text{ст}}$ $z_{\text{ст}}$ is the static shift of the center of rotor mass in the horizontal and vertical direction caused by deformations in the mounting groups; α is the dynamic tilt angle of the rotor journals; $\alpha_{\text{ст}}$ is the angle of inclination from the equilibrium position caused by the process load on the rotor; y_α, z_α are total dynamic displacements of the rotor mass center in the above directions; $f = Fz / Fy$ is the relationship of the vertical and horizontal components of the process load.

To determine the potential (Π) and kinetic (T) energies of the system under study, we use the following equalities:

$$\Pi = \frac{1}{2}C_y\left[y_a + y_{cm} - \delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm}) + \delta(\alpha_{cm})\sin\alpha_{cm}\right]^2 +$$

$$+\frac{1}{2}C_z\left[z_a + z_{cm} - \delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm}) + \delta(\alpha_{cm})\cos\alpha_{cm}\right]^2 -$$

$$-mg\left[z_a + z_{cm} + \delta(\alpha_{cm})\cos\alpha_{cm}\right];$$

$$T = \frac{1}{2}m\left(\dot{y}_a^2 + \dot{z}_a^2\right) + \frac{1}{2}A\omega^2 .$$

Here, A is polar moment of inertia, ω is the angular velocity of a rotating rotor.

It should be borne in mind that the acting forces are not potential. Moreover, the generalized forces to which the previously introduced coordinates y_α, z_α, α correspond, take the form:

$$Q_{y_a} = F_y,\ Q_{z_a} = -F_z,$$

$$Q_\alpha = F_y \begin{Bmatrix} \left[\delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})+\delta'(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})\right]+ \\ +f\left[\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})-\delta'(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})\right] \end{Bmatrix}.$$

Here, $\delta'(\alpha+\alpha_{cm})$ is α angle derivative of radial clearance; $f=\dfrac{F_z}{F_y}$ is the relationship of vertical and horizontal components of the process load.

Under these conditions, equations expressing the dynamics of the system can be written as:

$$\begin{cases} m\ddot{y}_a + C_y\cdot y_a - C_y\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm}) + C_y\delta(\alpha_{cm})\sin\alpha_{cm} = 0, \\ m\ddot{z}_a + C_z\cdot z_a - C_z\delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm}) + C_z\delta(\alpha_{cm})\cos\alpha_{cm} = 0, \\ mg\left[\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})-\delta'(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})\right]- \\ -C_y\left[y_a-\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})+\delta(\alpha_{cm})\sin\alpha_{cm}\right]\times \\ \times\left[\delta'(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})+\delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})\right]+ \\ +C_z\left[z_a-\delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})+\delta(\alpha_{cm})\cos\alpha_{cm}\right]\times \\ \times\left[\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})-\delta'(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})\right]= \\ =F_y\begin{Bmatrix} \left[\delta(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})+\delta'(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})\right]+ \\ +f\left[\delta(\alpha+\alpha_{cm})\sin(\alpha+\alpha_{cm})-\delta'(\alpha+\alpha_{cm})\cos(\alpha+\alpha_{cm})\right]\end{Bmatrix}. \end{cases} \tag{1}$$

Having examined in more detail the third equation of the system (1), we determine the angle of inclination α_{cm} considering $\alpha=0$. After transformations in the equations (1), we obtain a simpler system:

$$\begin{aligned} m\ddot{y}_a + C^y_{экв}\cdot y_a + C^{yz}_{экв}\cdot z_a = 0, \\ m\ddot{z}_a + C^z_{экв}\cdot z_a + C^{zy}_{экв}\cdot y_a = 0. \end{aligned} \tag{2}$$

An analysis of the equations (2) enables, in the first approximation of the values, to consider the elastic gapped-type support as a support with linear elastic characteristics (both in horizontal and vertical guides):

$$F_y = C^y_{экв}\cdot y_a + C^{yz}_{экв}\cdot z_a,\ F_z = C^z_{экв}\cdot z_a + C^{zy}_{экв}\cdot y_a.$$

Here,

$$C^y_{экв} = \frac{C_y\left[mg\cdot r+\cos^3\alpha_{cm}\left(\theta+f\cdot z\cdot z^2\cdot C_z\right)\right]}{\Delta},\ C^z_{экв} = \frac{C_z\left[mg\cdot r+\cos^3\alpha_{cm}\left(\theta+f\cdot z\cdot\theta^2\cdot C_y\right)\right]}{\Delta},$$

$$C^{yz}_{экв} = C^{zy}_{экв} = \frac{C_yC_z\cos^3\alpha_{cm}\cdot\theta\cdot z(\theta+f\cdot z)}{\Delta},$$

$$r=\delta^2(\alpha_{cm})+2\delta'(\alpha_{cm})^2-\delta(\alpha_{cm})\cdot\delta''(\alpha_{cm}),$$

$$\theta=\delta(\alpha_{cm})+\delta'(\alpha_{cm})\cdot tg\alpha_{cm},\ z=\delta(\alpha_{cm})\cdot tg\alpha_{cm}-\delta'(\alpha_{cm}),$$

$$\Delta = mg\cdot r+\cos^3\alpha_{cm}(\theta+f\cdot z)\cdot\left(C_z\cdot z^2+C_y\cdot\theta^2\right). \tag{3}$$

Research Results. In the system (3), the quantities $C^y_{экв}$, $C^z_{экв}$, $C^{yz}_{экв}$ express the equivalent stiffness characteristics of the gapped-type support.

To obtain the dependences of the equivalent stiffness characteristics of a non-linear mechanical system, we use the expansion of trigonometric functions in the system (1) taking into account higher-order summands. To determine the required values, we indicate:

$$y_a = A_y\cdot\sin\omega_1 t,\ z_a = B_z\cdot\cos\omega_1 t,$$

where B_z, A_y is the amplitude of the general oscillation of the rotor in the vertical and horizontal guides, respectively; ω_1 is the frequency of oscillations.

Thus,

$$C^{y}_{экв}=\frac{\omega_1}{\pi A_y}\int\limits_0^{\frac{2\pi}{\omega_1}}\Phi_y\left(y_a,z_a\right)\sin\omega_1 t dt,$$
$$C^{z}_{экв}=\frac{\omega_1}{\pi B_z}\int\limits_0^{\frac{2\pi}{\omega_1}}\Phi_z\left(y_a,z_a\right)\cos\omega_1 t dt. \qquad (4)$$

Here, $\Phi_y\left(y_a,z_a\right)$, $\Phi_z\left(y_a,z_a\right)$ are functions of y_a, z_a coordinates. These quantities can be determined through the system of differential equations (1).

Solving the system of equations (4) by integration methods, we represent the quantities in the form:

$$C^{y}_{экв}=\frac{C_y\left(mgr+C_z\cos\alpha_{cm}\cdot b\cdot z^2\right)}{\Delta}+\frac{C_y^2\cos\alpha_{cm}\cdot\psi\cdot\theta\cdot b^3\left(C_y^2\cdot\theta^2\cdot A_y^2+C_z^2\cdot z^2\cdot B_z^2\right)}{8\Delta^3},$$
$$C^{z}_{экв}=\frac{C_z\left(mgr+C_y\cos\alpha_{cm}\cdot b\cdot\theta^2\right)}{\Delta}+\frac{C_z^2\cos\alpha_{cm}\cdot\psi\cdot z\cdot b^3\left(C_y^2\cdot\theta^2\cdot A_y^2+C_z^2\cdot z^2\cdot B_z^2\right)}{8\Delta^3}. \qquad (5)$$

Here,

$$b=\cos^2\alpha_{cm}\left(\theta+f\cdot z\right),$$
$$\psi=\delta\left(\alpha_{cm}\right)+3\delta'\left(\alpha_{cm}\right)tg\alpha_{cm}-3\delta''\left(\alpha_{cm}\right)-\delta'''\left(\alpha_{cm}\right)tg\alpha_{cm},$$
$$\xi=\delta\left(\alpha_{cm}\right)tg\alpha_{cm}-3\delta'\left(\alpha_{cm}\right)-3\delta''\left(\alpha_{cm}\right)tg\alpha_{cm}+\delta'''\left(\alpha_{cm}\right).$$

Analyzing (5), we can talk about the interdependence not only between the stiffness of the housings in the rotor bearings C_y, C_z, but also the amplitudes A_y, B_z of its general vibrations.

It must be emphasized that stiffness $C^{yz}_{экв}$ is expressed as the relationship of rotor movements in the horizontal and vertical planes. In the case $C^{yz}_{экв}=0$ in the system (2), the equations will not be connected, which enables to consider all possible options. They are listed below.

— In the absence of changes in the radial clearance over time and $\alpha_{cm}=0$, we have $C^{z}_{экв}=C_z$. This option is typical for idle rotor operation. Moreover, the radial displacement of the rotor with respect to the support in the vertical guides is a very small amount, although of a higher order than the displacement in the horizontal guides.

— If we accept $z=0$, then $C^{z}_{экв}=C_z$. This situation is characteristic of the radial movement of the rotor relative to the support surrounded by a point through which the horizontal tangent passes to the trajectory of the relative movement of the journal center.

— When the value of the radial clearance in the bearings is zero, we have $C^{y}_{экв}=C_y$ and $C^{z}_{экв}=C_z$.

— If there are no changes in the radial clearance and $\alpha_{cm}=90^0$, then $C^{y}_{экв}=C_y$. This is possible if the rotor operates at under the process load with a selected gap, i.e., in horizontal rails, the radial displacement assumes small values in comparison to the vertical direction.

— When $\theta=0$, $C^{y}_{экв}=C_y$. In this case, the radial displacements of the rotor take place in the vicinity of the point through which the vertical tangent passes to the line of relative motion of the journal center.

— It is possible to accomplish the equality $\theta+f\cdot z=0$. This is true in the case of a radial displacement of the rotor with respect to the support surrounded by such a point through which a normal is drawn which coincides with the line of action of the resulting forces F_r and F_b. In this case, $C^{y}_{экв}=C_y$ and $C^{z}_{экв}=C_z$.

Consider the characteristics of the rotor on elastic supports the rigidity of which is equal to $C^{y}_{экв}$, $C^{z}_{экв}$. Let us denote ε the eccentricity of the rotor, and ω its angular velocity. We represent the oscillation equations in the form of the system:

$$\begin{cases} m\ddot{y}_a+C^{y}_{экв}\cdot y_a=m\cdot\varepsilon\cdot\omega^2\sin\omega t \\ m\ddot{z}_a+C^{z}_{экв}\cdot z_a=m\cdot\varepsilon\cdot\omega^2\cos\omega t \end{cases}. \qquad (6)$$

Solving the (6), we write the expressions for the oscillation amplitudes:

$$A_y = \frac{m \cdot \varepsilon \cdot \omega^2}{C^y_{экв} - m \cdot \omega^2}, \; B_z = \frac{m \cdot \varepsilon \cdot \omega^2}{C^z_{экв} - m \cdot \omega^2}. \tag{7}$$

Note that the summands $C^y_{экв}$, $C^z_{экв}$ in the denominators depend on the amplitudes A_y, B_z. We represent the graphically considered system (Fig. 2).

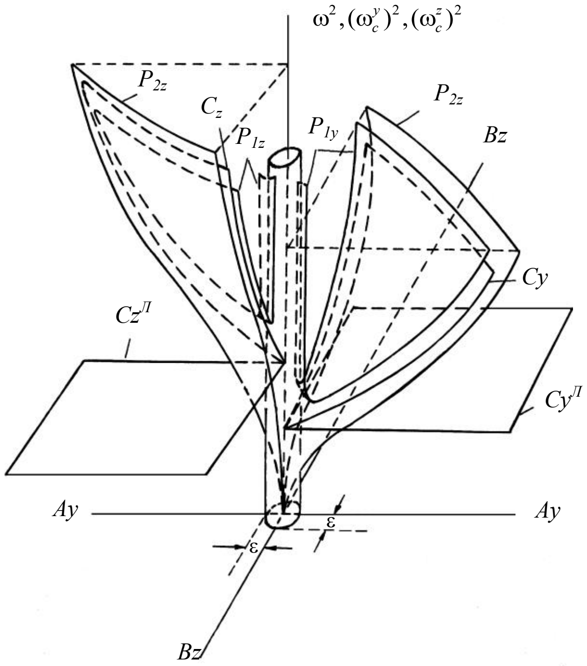

Fig. 2. Graphical representation of dynamic characteristics of absolute oscillations of the rotor in gapped-type supports

The surfaces P_{1y}, P_{2y}, P_{1z}, P_{2z} (resonant) and C_y, C_z (skeleton) are constructed under the assumption that the amplitudes A_y and B_z vary independently based on (7) and the equalities $C^y_{экв} = m\left(\omega^y_c\right)^2$, $C^z_{экв} = m\left(\omega^z_c\right)^2$.

The amplitudes of forced oscillations A_y, B_z correspond to the frequency of disturbing forces ω_0. Given this fact, to obtain expressions of the indicated amplitudes, we first construct the plane $\omega^2 = \omega_0^2$.

At the intersection lines of the constructed plane with the surfaces P_{1y}, P_{2y} and P_{1z}, P_{2z} there will be points with identical coordinates. These points are the desired amplitudes of the rotor oscillations.

Planes $C^Л_y$, $C^Л_z$ are the skeleton surfaces of a linear system. In this case, only the first summands are taken in the expressions for equivalent stiffness. They indicate that the frequencies of free vibrations of a linear system are independent of amplitudes.

Resonant surfaces represent rotor oscillations in horizontal guides, and skeleton surfaces – in vertical guides. They are built in various coordinate octants. The skeleton surfaces are elliptical paraboloids in shape. ω^y_c, ω^z_c represent the frequencies of free oscillations of the system in Fig. 2.

Discussion and Conclusions. Multiple points may be suitable for a single frequency. This means that several modes of oscillations including unstable ones are possible in the studied system. For the transition of a system from one stable mode of motion to another, some external impacts are necessary, which is characteristic of nonlinear systems.

As an example of the application of the dependences obtained, we can cite the problem on forced oscillations of the rotor due to its static imbalance.

Dependences presented in the paper (4) indicate that equivalent angular stiffness properties are interdependent not only through the stiffness characteristics C_y, C_z of the bodies, but also through the amplitudes A_y, B_z of general rotor oscillations.

As a result of the study on the linear characteristics of the mounting groups of the loaded rotor, the following results are obtained.

1. A dynamic model of a rotor rotating in elastic gapped-type supports is proposed. This model provides investigating the problem of determining the linear equivalent stiffness characteristics of mounting groups.

2. The system of equations is analyzed, and the possibilities of applying the formulas are listed. In particular, they can be used to determine rotor oscillations in the horizontal plane on shavers used to produce leather blanks.

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